



FIG. 23. The ratio of Young's modulus to the shear modulus of all of the elements considered in this review with the exception of the rare-earth elements. The horizontal dashed line represents the mean value of this ratio for these elements. Open points are estimated values.

The variation of  $Y/\mu$  for all the elements is shown in Fig. 23. The similarity between  $Y/\mu$  given in this plot and Poisson's ratio (Fig. 4) is quite close, as would be expected. The variation of  $Y/\mu$  as one proceeds from one element to the next is more strikingly illustrated by the  $Y/\mu$  ratio than by Poisson's ratio. The periodic dependence, such as is shown by the low values for the group IIIA metals and iron and its congeners, and the higher-than-average values for the group IA, IIA, IVA, VA, IB, VB, and VIB elements, is obvious.

An attempt was made to determine whether or not the ratio of  $Y/\mu$  has a crystal-structure dependence. It was found that the mean value was 2.64 for both the face-centered cubic, A1, and body-centered cubic, A2, metals; 2.62 for the diamond lattice, A4, elements; and 2.55 for the

hexagonal close-packed, A3, metals. Hence the data do not seem to reveal any dependence on the crystal structure.

The variation of  $Y/\mu$  for the rare earths is shown in Fig. 2c, where it is seen that the ratio decreases slightly with increasing atomic number. It should be observed, however, that the values for the ratio  $Y/\mu$  are more scattered than are the values for either Young's modulus or the shear modulus alone (Figs. 2a and 2b).

*Estimated Data.* First, a value for  $Y/\mu$  was estimated assuming that for a given element it was equal to the mean value obtained from the element's congeners. Second, an estimated value of  $Y/\mu$  was calculated from the estimated value of Poisson's ratio (see Section 5) by using Eq. (5.1). Third, these two estimated values were averaged to obtain the values shown in Table XIX and in Fig. 23. This procedure was used to estimate the values for phosphorus, scandium, arsenic, selenium, rubidium, strontium, technetium, cesium, osmium, polonium, francium, radium, and actinium. The estimated values for protactinium and neptunium for the first step were assumed to be equal to the mean value of thorium and uranium, and to the value of uranium, respectively. The other two steps of the procedure given above were followed exactly for estimating the value of  $Y/\mu$  for protactinium and neptunium. The estimated values for  $Y/\mu$  for the rare earths promethium, europium, thulium, and lutetium were calculated directly from the estimated values of Young's modulus (see Section 3) and the shear modulus (see Section 4).

The mean value of all the estimated  $Y/\mu$  ratios is 2.603, which shows that mean value of the experimental  $Y/\mu$  ratios would remain unchanged if the estimated values were included.

### 23. THE RATIO $(1 - \sigma)/(1 + \sigma)$

One of the steps in calculating the size factor involves the ratio  $(1 - \sigma)/(1 + \sigma)$  (see Section 29), and because of this, these ratios are listed in Table XX.

The average value of Poisson's ratio for the elements is 0.301. If this value is substituted into the ratio  $(1 - \sigma)/(1 + \sigma)$ , a value of 0.536 is obtained. The minimum and maximum values possible for this ratio are 0.333 for  $\sigma = 0.5$  and 1.000 for  $\sigma = 0$ . Furthermore, since  $\sigma$  is essentially a constant, the ratio  $(1 - \sigma)/(1 + \sigma)$  would be expected to be a constant also. The average for all of the experimental values is  $0.543 \pm 0.102$ . The values of beryllium and boron are anomalously large, and if they are excluded from the average, a value of  $0.533 \pm 0.083$  is obtained. The latter value is thought to be a more reliable average than the former.